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The Delivery of Market Timing Services: Newsletters versus Market Timing Funds*

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We examine delivery systems that disseminate market timing information either through newsletters or by setting up timing funds in which investors can invest. Absent market imperfections, both systems produce the same result. With restrictions on borrowing, or with other nonlinearities, the newletter system is superior. This result does not depend on the cost of obtaining information or uncertainty about, or the manipulation of, the quality of the information. Institutional restrictions on borrowing, and preferences that lead to nonlinear responses to information signals, provide one explanation for the plethora of market timing newsletters and the paucity of market timing funds. © 1990 Academic Press, Inc.

INTRODUCTION

Market timing information relates to performance of equities relative to risk-free assets. Merton (1981) has shown that such generalized information can lead to large gains. Market timing delivery systems attempt to package and to deliver market timing information. In spite of the potential gains from market timing, there are no mutual funds that specialize in

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market timing, and market timing information is mostly delivered via newsletters. This paper provides a new explanation for this phenomenon.

Following the seminal work by Grossman and Stiglitz (1980), the Admati–Pfleiderer model (Admati, 1985; Admati and Pfleiderer, 1987, 1986, 1984) is an equilibrium theory emphasizing the behavior of the suppliers of information. As applied to the delivery of market timing information, their model explains the paucity of market timing funds and the prevalence of newsletters as resulting from: (i) the desire of information suppliers to control the quality of the information sold (introduce noise into their signals), and/or (ii) the use by investors of private information in conjunction with the acquired information which would preclude indirect sale of information via portfolio management services.

The present paper reveals that institutional friction, namely restricted borrowing, is sufficient to render market timing funds inferior to newsletters even without strategic play by suppliers and even without private investor information. More fundamentally, the superiority of direct sale of information (newsletters) over indirect delivery via timing funds is attributed to nonlinearities in the relation of the optimal portfolio to the information signal (in either the objective function or the constraints). This explanation is not inconsistent with that of the Admati–Pfleiderer model; thus the model provides an additional independent explanation for the observed phenomenon.

We envision two delivery systems, one in which market analysts with timing information set up timing funds (the indirect sale of information) and one in which market analysts with timing information sell their information in the form of newsletters (the direct sale of information). Addressing the issue of the most efficient delivery system we find that, in the absence of market imperfections, both systems give identical results. In the presence of restrictions on borrowing (or other nonlinearities), the newsletter delivery system is superior.

In Section I we present the benchmark model which is an extension of Grant (1977) and Dybvig and Ross (1985). Section II derives the unconstrained optimal use of timing information, and provides a benchmark against which to measure the performance of various information delivery systems. A description of the timing funds and newsletter delivery systems is given in Section III, and the evaluation of their performance under ideal conditions is presented in Section IV. In Section V we assess the effects of borrowing constraints and nonlinear response functions on the efficacy of the delivery system. A summary concludes the paper.

I. THE MODEL

Investors are restricted to two asset classes: a risk-free asset and the risky market portfolio. Investors are then partitioned into two classes: passive and active.

Passive investors allocate their funds between the risk-free asset and a market-index fund that is always fully invested in the market portfolio. They make no use of private information, either directly or indirectly. While the allocation may change from time to time on the basis of personal circumstance, changes are uncorrelated with subsequent rates of return.

Active investors use private information. When using information directly, active investors shift their savings between the risk-free asset and the market-index fund on the basis of signals about the future relative performance of the market portfolio.

When using information indirectly, active investors allocate their risk-bound savings to one or more timing funds. Timing funds shift their capital between the market portfolio and the risk-free asset on the basis of private signals obtained by the fund managers.

For simplicity, assume that a dollar's worth of the index portfolio will sell in one period for e^x, and that σ_x^2 is the constant variance of x. Investors that choose not to use market timing information assume that the expected value of x is $r + \pi$, where r is the (known) risk-free rate and π is the market risk premium. The reward to volatility ratio (Sharpe's (1966) measure), π/σ, is taken by the uninformed investors to be the known (assumed constant) market price of risk.

Investors that choose to become informed about the market (henceforth timers) assume that the random component (ε) in the rate of return, $x = r + \pi + \varepsilon$, can be (imperfectly) anticipated from a proprietary signal. The risk-free rate is assumed to be zero (this is unimportant) and ε is normal with mean zero and variance of σ_ε^2. The timer invests a unit amount under a constant absolute risk aversion utility function, $u(\omega) = -\exp[-A\omega]$. We assume that there are no principal–agent problems and that the volume of transactions that is associated with investors who engage in market timing is difficult to distinguish from hedging, liquidity trading, and security analysis.

Suppose that a timer obtains a calibrated signal[1] (s) with a quality ρ, so that now ε is conditionally distributed by $\varepsilon|s \sim N(s,(1 - \rho^2)\sigma_\varepsilon^2]$. Since the

[1] A calibrated (often referred to as a "rational expectations") signal is derived from a raw signal (s') that is generally distributed by $s' = f(\varepsilon) + \eta$, where η is white noise. The calibration is aimed at ridding the raw signal of the biases introduced by both f and η. In a simple case, $s'|\varepsilon \sim N(\varepsilon, \sigma_\eta^2)$, and the quality of the signal is given by the correlation coefficient between s' and ε: $\rho^2 = \sigma_\eta^2/(\sigma_\eta^2 + \sigma_\varepsilon^2)$. With the calibrated, unbiased and efficient, signal (Theil, 1971), $s = \rho^2 s'$, so that ε will be distributed as in the text. We assume that the signals received by the timers are uncorrelated *conditional* on ε. (This means that they will be unconditionally correlated since they all provide some information about ε.) This assumption is unimportant for the results herein and serves to simplify the mathematics. The only qualification is that if all signals were perfectly, conditionally, correlated then we would have in effect only one signal. In this case the two delivery systems will give identical results.

total return on the market is assumed $x = \pi + \varepsilon$, the conditional distribution of x is $x|s \sim N[\pi + s, (1 - \rho^2)\sigma_\varepsilon^2]$.

A timer with a signal s will invest $\gamma(s)$ in the market and the remainder, $1 - \gamma(s)$, in the risk-free asset. The return on the portfolio (with a risk-free rate of zero) is just $\gamma(s)x$. $\gamma(s)$ is chosen to maximize the expected negative exponential utility function, $U = E\{-\exp[-A(1 + x)|s]\}$, Hence, with the assumption of normality,

$$\gamma^*(s) = \frac{E(x|s)}{A\ \mathrm{Var}(x|s)} = \frac{\pi + s}{A(1 - \rho^2)\sigma_\varepsilon^2}, \tag{1}$$

as in Tobin (1958), Grant (1977), and Dybvig and Ross (1985).[2]

II. THE OPTIMAL UNCONSTRAINED USE OF INFORMATION

Consider a portfolio manager who receives n conditionally independent signals. Our purpose is to establish an ideal benchmark by which to measure performance under the two information delivery systems. Denote the signals $s_1, \ldots, s_n$, with the known qualities, $\rho_1, \ldots, \rho_n$, respectively. The distribution of the market return given all signals is given in Proposition 1.

PROPOSITION 1. *The distribution of x conditional on $s_1, \ldots, s_n$ is given by*

$$(x|s_1, \ldots, s_n) \sim N[\pi + S, (1 - R^2)\sigma_\varepsilon^2], \tag{2}$$

where the summary signal, S, is a sufficient statistic for the set $\{s_i\}$, and computed as the weighted average: $S = \sum w_i s_i$. The quality, R^2, of the summary signal (the squared correlation coefficient between S and ε) is also the weighted average of the individual signal qualities: $R^2 = \sum w_i \rho_i^2$. The weights $\{w_i\}$ are the optimal scaling factors, determined by the relative quality of the signals:

$$w_i = \left(\frac{1}{1 - \rho_i^2}\right) \Big/ \left(1 + \sum \frac{\rho_j^2}{1 - \rho_j^2}\right). \tag{3}$$

[2] The assumption of linearity in the signal comes from assuming negative exponential utility and normality. One reason to begin with the assumption of linearity is that it is used extensively in the literature. Another is that it significantly simplifies the model. Most importantly, however, we intend to show how linearity affects the two delivery systems under consideration. We do this by relaxing this assumption in Section V.2. Of course, we would be surprised if response functions were in fact linear, which makes Proposition 7 all the more interesting.

(The proof of Proposition 1 is in the Appendix.) Note that the signal scaling factors add up to more than 1.0 as,

$$\sum w_i = \left(n + \sum \frac{\rho_i^2}{1-\rho_i^2}\right) \bigg/ \left(1 + \sum \frac{\rho_j^2}{1-\rho_j^2}\right) > 1.$$

Intuition for the scaling can be gained from the way a raw signal is calibrated (see footnote 1). Each raw signal, s_i', is first discounted by the factor ρ_i^2 to account for its noise. Using multiple signals with independent forecasting errors, we note that some of this noise will be offset in a portfolio effect. Hence, we should undo some of the original discounting.

To illustrate, with forecasters of equal ability ($\rho_i = \rho$) we would have

$$R^2 = \frac{n\rho^2}{1 + (n-1)\rho^2}; \qquad \lim_{n\to\infty} R^2 = 1,$$

$$w_i = \frac{1}{1 + (n-1)\rho^2}; \qquad \lim_{n\to\infty} w_i = 0,$$

$$\sum w_i = \frac{n}{1 + (n-1)\rho^2}; \qquad \lim_{n\to\infty} \sum w_i = \frac{1}{\rho^2}.$$

With two independent forecasts of equal quality ($\rho_1 = \rho_2 = \rho$) we would have

$$w_1 = w_2 = \frac{1}{1+\rho^2} = w; \qquad S = \frac{s_1 + s_2}{1+\rho^2}; \qquad R^2 = \frac{2\rho^2}{1+\rho^2}.$$

If, for example, $\rho^2 = \frac{1}{2}$, then each of the two signals is weighted by $s = \frac{2}{3}$, and the weights sum to $\frac{4}{3}$.

The optimal market position of a timer who possesses all signals $s_1, \ldots, s_n$ follows directly from Proposition 1 and Eq. (1):

$$\gamma^*(s_1, \ldots, s_n) = \frac{\pi + S}{A(1-R^2)\sigma_\varepsilon^2}, \tag{4}$$

which is the multiple signal analog to (1). This is our benchmark. It will be useful to continue our example with n signals of equal quality. Here the investor's (linear) response function under the assumptions of (4) will apply to the sum of the individual signals; i.e., the optimal position in the market will be $\gamma^*(s_1, \ldots, s_n) = a + b \sum s_i$, where $a = \pi/[A(1-\rho^2)\sigma_\varepsilon^2]$ and $b = a/\pi$.

III. DELIVERY SYSTEMS

Consider two delivery systems. Under both systems the i^{th} signal from the set $\{s_i\}$ is obtained by the i^{th} timer. Under the *newsletter delivery system* (direct sale of information), active individual investors obtain the information from timers through newsletters, and use it to make timing decisions. Under the *fund delivery system* (indirect sale of information), each timer sets up a separate fund. Individual investors then diversify among the many funds. In both systems, individual investors may borrow or lend at the risk-free rate and also may invest directly in a (passive) market-index fund.

Under the newsletter delivery system, we will assume that an investor allocates W to active timing, W_y to the market-index fund, and the remainder, $1 - W - W_y$, to the risk-free asset (see Fig. 1*A*). We split the risky investment into W and W_y artificially in order to facilitate comparisons between the delivery systems.

Under the fund delivery system, investors allocate W_i to the i^{th} timer fund, W_x to the market-index fund, and the remainder, $1 - \Sigma W_i - W_x$, to the risk-free asset. (see Fig. 1*B*). The notation for each system is summarized below.

1. *Newsletter Delivery System*

s_i = signal received by i^{th} timer and reported in i^{th} newsletter.

W = investor allocation to his own timing fund.

$\gamma(s_1, \ldots, s_n)$ = proportion of own timing fund's assets (W) invested in the market portfolio.

W_y = investor allocation to the market-index fund

$\gamma_N(s_1, \ldots, s_n) = W\gamma(s_1, \ldots, s_n) + W_y$ = total investment in the market portfolio.

2. *Fund Delivery System*

s_i = signal received by the i^{th} timer who uses it in his own timing fund.

W_i = investor allocation to the i^{th} timing fund.

$\gamma_i(s_i)$ = proportion of i^{th} fund's assets (W_i) invested in the market portfolio.

W_x = allocation to the market-index fund.

$\gamma_T(s_1, \ldots, s_n) = \Sigma W_i\gamma_i(s_i) + W_x$ = total investment in the market portfolio.

IV. EVALUATING PERFORMANCE WITHOUT CONSTRAINTS

Consider first the outcomes under the two systems with the model assumptions. In Section V we relax the assumption of unrestricted bor-

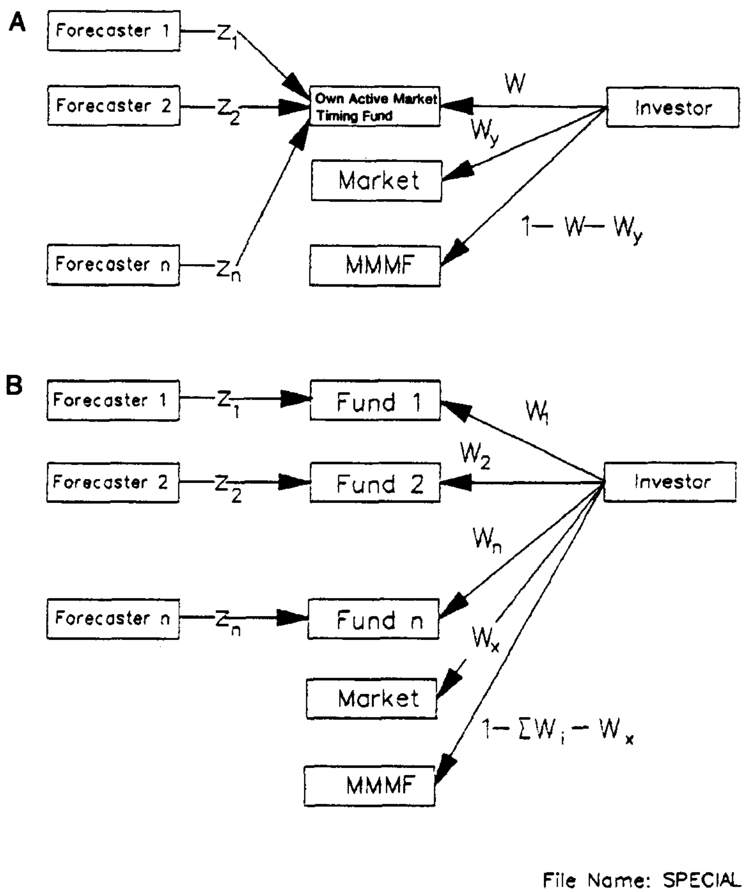

FIG. 1. (A) Newsletter delivery system. (B) Fund delivery system.

rowing and of a linear reaction function to the signal (negative exponential utility).

The following two propositions specify what can be achieved through the two delivery systems. We therefore make no assumptions about the decision rules of the various funds except to say that the proportion invested in the market is a function of the signal. We then explore which system, when using its best decision rules, will produce the best results for the investor.

Proposition 2 establishes that the optimal use of information can be achieved under the newsletter system. (Proposition 2 is true by definition, since without constraints, one just sets the reaction function at the optimum. We state this as a proposition to contrast it with the less obvious Proposition 3.)

PROPOSITION 2 (Tautology). *Under the newsletter delivery system the investor's overall position in the market is equal to the optimal position if* γ_N *is chosen such that*

$$\gamma_N(s_1, \ldots, s_n) = \gamma^*(s_1, \ldots, s_n).$$

As an aside, note that the portfolio separation property is preserved. A single fund collecting all newsletters can serve all investors, even when they exhibit different degrees of risk aversion (A). The fund announces

$$\gamma_N = a + bS$$

and the investor chooses W and W_y such that

$$W = \frac{1}{bA(1 - R^2)\sigma_\varepsilon^2}; \qquad W_y = \frac{b\pi - a}{bA(1 - R^2)\sigma_\varepsilon^2}. \tag{5}$$

Note that W and W_y can be chosen without regard to the signals themselves.

Proposition 3 states that the optimal use of information can also be achieved under the fund delivery system.

PROPOSITION 3. *Under the fund delivery system the investor's overall position in the market is optimal, i.e.,* $\gamma_T(s_1, \ldots, s_n) = \gamma^*(s_1, \ldots, s_n)$,

if:

$$\gamma_i(s_i) = a_i + b_i s_i,$$

and if: the allocation to risky investments satisfies

$$W_i = \frac{w_i}{b_i A(1 - R^2)\sigma_\varepsilon^2}; \qquad W_x = \frac{\pi - \sum (a_i/b_i)w_i}{A(1 - R^2)\sigma_\varepsilon^2}, \tag{6}$$

where w_i and R depend only on $\rho_1, \ldots, \rho_n$.

Proof.

$$\gamma_T(s_1, \ldots, s_n) = W_x + \sum W_i\gamma_i(s_i) = \tau^*(s_1, \ldots, s_n). \quad \text{Q.E.D.}$$

As in the newsletter case, portfolio separation prevails and investors with different degrees of risk aversion can be served by the same set of timing funds.

V. MARKET IMPERFECTIONS: CONSTRAINTS AND OTHER NONLINEARITIES

In the model so far, information disseminated through timing funds is as valuable as information disseminated through newsletters. This result, however, depends on two key assumptions: (1) there are no restrictions on borrowing and short positions in the market, and (2) the optimal amount invested in the market is linear in the signal. If we relax either of these assumptions, information disseminated through newsletters is more valuable than information disseminated through timing funds.

1. *Restrictions on Borrowing*

We now introduce borrowing restrictions both on funds and on investors. Assume that investors can borrow a proportion of net assets equal to B_I. Thus investors can invest $1 + B_I$ dollars for every dollar of net assets. Likewise, funds can borrow a proportion of net assets equals to B_F, and thus can invest $1 + B_F$ dollars in the market for every dollar of net assets. Of course, neither investors nor funds have to borrow the full amount available.

With borrowing restrictions, for the reason that there is a greater potential need to borrow under the fund delivery system, a newsletter delivery system would be superior. To this end we show that (i) under the newsletter delivery system, one can replicate the best outcome achievable under the fund delivery system, but (ii) under the fund delivery system, one cannot replicate the best newsletter outcome. Taken together these two propositions imply that the newsletter system must be strictly superior. A discussion of the results will follow.

PROPOSITION 4. *With borrowing restrictions on both individuals and funds, the newsletter delivery system outcome can replicate the best fund delivery system.*

Proof. The best outcome when diversifying among timing funds is found by choosing $\gamma_i(s_i)$, W_i, and W_x so as to maximize expected utility, subject to borrowing constraints on both the investor and the fund:

$$\gamma_T(s_1, \ldots, s_n) = \sum W_i\gamma_i(s_i) + W_x$$

subject to:

$$W_x + \sum W_i \leqq 1 + B_I,$$
$$\gamma_i(s_i) \leqq 1 + B_F, \qquad \text{for all } i.$$

The newsletter outcome is defined by

$$\gamma_N(s_1, \ldots, s_n) = W\gamma(s_1, \ldots, s_n) + W_y,$$

subject to:

$$W + W_y \leqq 1 + B_I,$$

$$\gamma(s_1, \ldots, s_n) \leqq 1 + B_F.$$

If investors were to set $W = 1 + B_I$, $W_y = 0$, and have their newsletter (single) timing fund invest $\gamma(s_1, \ldots, s_n) = [\sum W_i\gamma_i(s_i)]/(1 + B_I)$ in the market, then

$$\begin{aligned} \gamma_N(s_1, \ldots, s_n) &= W_{\gamma}(s_1, \ldots, s_n) + W_y \\ &= \sum_i \gamma_i(s_i) + W_x = \tau_T(s_1, \ldots, s_n) \end{aligned}$$

without violating borrowing constraints, since $W = 1 + B_I$ and

$$\begin{aligned} \gamma(s_1, \ldots, s_n) &= \frac{\sum W_i\gamma_i(s_i) + W_x}{1 + B_I} \\ &\leqq \frac{\sum W_i(1 + B_F) + W_x}{1 + B_I} \leqq \frac{(1 + B_F)(1 + B_I)}{1 + B_I} \qquad \text{Q.E.D.} \end{aligned}$$

Thus, with borrowing restrictions the newsletter system can replicate any fund delivery system outcome and, in particular, can replicate the optimal fund delivery system. The reverse is not true as the following proposition indicates.

PROPOSITION 5. *With borrowing restrictions on both individuals and funds, the fund delivery system cannot replicate the best newsletter outcome.*

Proof. Assume that the fund delivery system can replicate the optimal newsletter outcome. That is, suppose there exist $\gamma_i(s_i)$, W_i, and W_x such that

$$\sum W_i\gamma_i(s_i) + W_x = \gamma_N{}^*(s_1, \ldots, s_n),$$

where $\gamma_N{}^*$ is the optimal newsletter outcome with borrowing constraints. We know that the distribution of x depends only on $S = \sum w_i s_i$, and not on the individual s_i. Thus, we can write $\gamma_N{}^*(s_1, \ldots, s_n) = f(S)$, regardless of the nature of the constraints. Substituting yields

$$\sum W_i\gamma_i(s_i) + W_x = f(S).$$

Taking the total differential with respect to the s_i yields

$$\sum W_i\gamma_i'(s_i)ds_i = f'(S)\sum w_i ds_i$$

which implies

$$W_i\gamma_i'(s_i) = f'(S)w_i, \qquad \text{for all } i. \qquad \text{Q.E.D.} \quad (7)$$

Equation (7) will hold only if (i) $\gamma_i(s_i)$ does not depend on s_i, but only on S, and is therefore linear in s_i. (Varying s_i while holding S constant must leave $\gamma_i(s_i)$ unchanged.) Also, (ii) $f'(S)$ does not depend on S, but only on s_i. Therefore, $f(S)$ must be linear in S. Thus the timer outcome can replicate the optimal newsletter outcome only if $\gamma_i(s_i)$ is linear in s_i and $f(S)$ is linear in S. With borrowing restrictions neither the $\gamma_i(s_i)$ nor $f(S)$ are linear. Combining Propositions 4 and 5 we get:

PROPOSITION 6. *With borrowing restrictions on both individuals and funds, the newsletter delivery system is superior to the fund delivery system.*

Individual investors can make use of newsletters in one of two ways. An investor can subscribe to newsletters and shift between the market and the safe asset according to the signals received. Alternatively, the investor can incorporate a timing fund. The timing fund would receive all the signals and shift assets between the market and the safe asset. The individual investor will then put some money into this fund. Since this latter arrangement is always a possibility, the imposition of individual constraints while leaving funds unconstrained would not result in any effective constraint at all.[3] That is, the fund could borrow in place of the individual. In such a case, the optimal result could be attained. This is true in both systems since under the fund system the funds could also borrow in place of the individual. Thus imposing individual constraints (without fund constraints) would allow the optimal result to be attained in both the newsletter system and the fund system.

Note also that if funds are constrained but individuals are not, then we get a similar result. That is, in either system the individual can get arbitrarily close to the optimal result. For example, in the newsletter case, the response function $\gamma_N = a + bS$ is chosen such that a is less than 1.0 and

[3] Again, we are making no assumptions about investor or individual behavior. Rather we are exploring possibilities in order to determine which system has the greater potential for performance for the investor. Thus in the newsletter system with investor constraints but no fund constraint the investor would do best by incorporating a fund to receive the signals and letting the fund borrow. Of course, many investors could be pooled into one fund.

b is very small, making it very unlikely that the fund will have to borrow. The investor then chooses values for W (investment in the newsletter fund) and W_y (investment in the passive market index fund), accordingly. Since b is small, this will entail a large W, compensated with a large short position (W_y) in the index fund. The net result, $W + W_y$, as can be seen from Eq. (5), will be large. That is, the individual will have to borrow a large amount. In effect, the investor is borrowing in place of the fund. The result can be made arbitrarily close to the optimum through the choice of a sufficiently small b. The same can be said about the fund delivery system, where the choice of sufficiently small b_i will result in an outcome arbitrarily close to the optimum.

Thus, the results given in Propositions 4–6 require that there be borrowing constraints on both individuals and funds. Intuitively, the results follow from the fact that under the newsletter system, extreme signals are netted out *before* they affect investment decisions, while in the fund system the offsetting of signals is affected via the netting out of investment positions. If investment positions are restricted, then the offsetting of signals will be distorted. A very optimistic signal may result in a constrained investment position. On the other hand, the average signal (S) used by the newsletter system, which may sometimes be unrestricted, is equivalent to a fund system where "unused constraints" may be traded.

To illustrate, consider the two signal case. Let us suppose that under the fund system there are two funds, each receiving a signal s_i. Let the reaction functions of these funds be identical:

$$\gamma(s_1) = .8 + 20s_1; \qquad \gamma(s_2) = .8 + 20s_2.$$

Let us further assume the following parameters:

$\rho_1^2 = \rho_2^2 = .1$ (identical signal qualities);
$A = 3$ (individual risk aversion);
$\pi = .08$, $\sigma_\varepsilon = .2$ (market mean and standard deviation).

The combined signal is then the following linear combination of the s_i:

$$S = (\tfrac{10}{11})s_1 + (\tfrac{10}{11})s_2.$$

The propositions state that with borrowing constraints on both individuals and funds, a newsletter system can do better than the fund system. To see that such a dominant newsletter system exists, let us assume that the investor incorporates a timing fund that receives both signals from newsletters and shifts funds according to the following function:

$$\gamma_N(s_1, s_2) = .8 + 11S = .8 + 10(s_1 + s_2).$$

Now let us suppose that neither fund can borrow. In this case the timing funds will bump into the borrowing constraint whenever s_1 or s_2 is greater than .01. The newsletter fund will bump into the constraint only when $s_1 + s_2 > .02$. It is easy to see that timing funds can be constrained in cases where the newsletter fund is not, for example, if $s_1 = .02$ and $s_2 = -.01$. The converse is not true, however. If $s_1 + s_2 > .02$, then either s_1 or s_2 must be greater than .01.

In the example, if the funds are unconstrained then the optimal result can be obtained with either system, even if individuals are constrained. To attain the optimal result in the fund system, the individuals would invest .463 in each fund and put .074 in a passive market fund. To attain the optimal result in the newsletter system, the individual would put .926 in the newsletter fund and .074 in a passive market fund. In fact, for any $(A; \pi; \sigma_\varepsilon)$, it is possible to find a_i and b_i (in the fund case) and a and b (in the newsletter case) such that the investor will not have to borrow. In effect, the funds can borrow for the investor.

If the funds are constrained but the individual is not, then by decreasing the b_i (in the fund case) and the b (in the newsletter case), we can reduce arbitrarily the probability of bumping into constraints and thus approach the optimum. For example, in the fund case, we could have $\gamma(s_i) = .8 + 10s_i$. This would make it less likely to bump into borrowing constraints, since it would happen only if s_1 or s_2 were greater than .02 (rather than .01 above). Investors, however, will then be required to borrow. This escape is not possible if both individuals and funds are constrained.

With constraints on both individuals and funds, it is always possible to find a newsletter fund configuration that is superior to the fund configuration, as we did in the exercise above. Magnitudes of the individual and fund constraints do not affect the superiority, as long as we compare systems on a *ceteris paribus* basis. Partially relaxing either the fund or the individual constraints will lessen the magnitude of this superiority. But as long as both constraints are finite, there exists a probability (however small) that the fund constraint will be encountered even when the individual has levered up to the maximum allowed by the individual constraint. Thus, corresponding to every market timing fund configuration and the associated individual and institutional borrowing constraints, there exists a newsletter fund configuration that does better.

In reality, we would expect to find institutional and legal constraints on both funds and individuals.

2. *Nonlinear Response Functions*

The assumptions in Section IV lead to an *unconstrained* optimal market position that was linear in S. (This came from normality and the negative exponential utility function.) Other utility functions will not have this property. For example, the unconstrained optimum for quadratic utility is

$$f(s_1, \ldots, s_n) = \frac{\pi + S}{Q[(\pi + S)^2 + (1 - R^2)\sigma_\varepsilon^2]},$$

where Q is a measure of risk aversion. Consider the following proposition for arbitrary reaction functions.

In the absence of constraints, the newsletter delivery system outcome can replicate any arbitrary reaction function $f(s_1, \ldots, s_n)$. Simply let $\gamma_N(s_1, \ldots, s_n) = f(s_1, \ldots, s_n)$. In particular, the newsletter delivery system can replicate the optimal reaction function, even if nonlinear.

This is not the case for the fund delivery system. Proposition 5 tells us that, in the absence of constraints, the fund delivery system can replicate an arbitrary reaction function $f(s_1, \ldots, s_n)$ if and only if $f(s_1, \ldots, s_n)$ is linear (affine) in the s_i. As a result we have the following proposition:

PROPOSITION 7. *In the absence of constraints and with a nonlinear optimal response function, the newsletter delivery system outcome is superior to the fund delivery system outcome.*

Thus, nonlinearities serve a function similar to borrowing constraints in that they cause the newsletter delivery system to be superior.

VI. CONCLUSIONS

We investigated two market timing information delivery systems, one in which timers set up timing funds and the other in which timers sell their information through newsletters. In the absence of market imperfections, we found that both systems produce the same result.

With restrictions on borrowing, or with nonlinear response functions, we found the newsletter delivery system to be superior. In practice, there are borrowing restrictions and other nonlinearities. We thus hypothesize that the newsletter delivery system leads to superior outcomes. This may be one explanation for the plethora of market timing newletters and the paucity of pure timing funds.

Appendix: PROOF OF PROPOSITION 1

PROPOSITION 1. The distribution of x conditional on $s_1, s_2, \ldots, s_n$ is given by

$$(x|s_1, \ldots, s_n) \sim N[\pi + S, (1 - R^2)\sigma_\varepsilon^2],$$

where

$$S = \sum w_i s_i; \qquad R^2 = \sum w_i \rho_i^2; \qquad w_i = \left(\frac{1}{1 - \rho_i^2}\right) \Big/ \left(1 + \sum \frac{\rho_j^2}{1 - \rho_j^2}\right)$$

Proof. Suppose that, as in footnote 1 of the text,

$$s_i = \sum \rho_i^2 s_i'; \qquad s_i'|\varepsilon \sim N[\varepsilon, \sigma_\varepsilon^2/k_i]; \qquad k_i = \frac{\rho_i^2}{1 - \rho_i^2};$$

then, using Bayes formula,

$$f(\varepsilon|s_i', \ldots, s_n') = \frac{f(s_1', \ldots, s_n'|\varepsilon)f(\varepsilon)}{f(s_1', \ldots, s_n')},$$

where f refers to the probability density function of the corresponding random variable.

Since we want to derive the distribution of ε given $\{s_i\}$, we will use

$$f(S|s_i', \ldots, s_n') \propto f(s_1', \ldots, s_n'|\varepsilon)f(\varepsilon),$$

where $\propto$ means "proportional to."

$\{s_i'\}$are conditionally independent normals, and ε is unconditionally normal. Hence, we can write

$$f(\varepsilon|s_i', \ldots, s_n') \,\alpha\, f(s_1'|\varepsilon)f(s_2'|\varepsilon) \cdots f(s_n'|\varepsilon)f(\varepsilon)$$

$$\propto \exp\left[\sum \frac{(s_i' - \varepsilon)^2}{2\sigma_\varepsilon^2/k_i} + \left(-\frac{\varepsilon^2}{2\sigma_\varepsilon^2}\right)\right]$$

$$\propto \exp\left\{\frac{-1}{2\sigma_\varepsilon^2}\left[\varepsilon^2 + \sum k_i\left(s_i' - \varepsilon\right)^2\right]\right.$$

$$\propto \exp\left\{\frac{-1}{2\sigma_\varepsilon^2}\left[\varepsilon^2\left(1 + \sum k_i\right) - 2\varepsilon\left(\sum s_i' k_i\right) + U\right]\right\}$$

$$\propto \exp\left\{\frac{\left(-1 + \sum k_i\right)}{2\sigma_\varepsilon^2}\left[\left(\varepsilon + \frac{\sum s_i' k_i}{1 + \sum k_i}\right)^2 + V\right]\right\}$$

$$\propto \exp\left\{\frac{-1}{2\sigma_\varepsilon^2(1 + \sum k_i)^{-1}}\left(\varepsilon - \frac{\sum s_i' k_i}{1 + \sum k_i}\right)^2\right\},$$

where U and V are terms that do not include s. This is enough to conclude that

$$\varepsilon|s_1', \ldots, s_n' \sim N\left[\frac{\sum s_n' k_i}{\left(1 + \sum k_i\right)}, \sigma_\varepsilon^2(1 + \sum k_i)^{-1}\right]$$

which is what we need, since $s_i = \rho_i^2 s_i'$, $k_i = \rho_i^2/(1 - \rho_i^2)$, $x = \pi + \varepsilon$, and

$$1 - R^2 = \frac{1}{1 + \sum k_i}; \qquad R^2 = \frac{\sum k_i}{1 + \sum k_i}.$$

R is indeed the correlation between S and ε. Consider that

$$[\mathrm{Corr}(S, \varepsilon)]^2 = \frac{[\mathrm{Cov}(S, \varepsilon)]^2}{\mathrm{Var}(S)\mathrm{Var}(\varepsilon)}; \qquad S = \sum w_i s_i,$$

and hence,

$$\mathrm{Cov}(S, \varepsilon) = \Sigma w_i \rho_i^2 \sigma_\varepsilon^2;$$
$$\mathrm{Var}(S) = E[(\Sigma w_i s_i^2)^2] = \Sigma w_i^2 \rho_i^2 \sigma_\varepsilon^2 + \Sigma\Sigma w_i w_j \mathrm{Cov}(s_i, s_j),$$

where

$$\begin{aligned}\mathrm{Cov}(s_i, s_j) &= E_\varepsilon E[(s_i - \rho_i^2\varepsilon)(s_j - \rho_j^2\varepsilon) + \varepsilon(\rho_i^2 s_j + \rho_j^2 s_i) - \varepsilon^2 | \varepsilon] \\ &= E_\varepsilon[\rho_i^2\rho_j^2\varepsilon^2] = \rho_i^2\rho_j^2\sigma_\varepsilon^2.\end{aligned}$$

Thus,

$$[\mathrm{Corr}(S, \varepsilon)]^2 = \frac{(\Sigma w_i \rho_i^2)^2}{\Sigma\Sigma w_i w_j \rho_i^2 \rho_j^2} = \frac{\Sigma k_i}{1 + \Sigma k_i}. \qquad \text{Q.E.D.}$$

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